Unit - I

Soln. of Equations and Eigenvalue Problems

Part A Qns

1. State the Order of convergence & Condition (criterion) of Convergence of Newton's Method. Nov/Dec 14

Ans:

Order of Convergence is 2.

Condition for convergence is

$$|f(x) f''(x)| < |f'(x)|^2.$$

2. Find an iterative formula to find $\sqrt{N}$, where N is positive number by Newton's Method.

Ans:

Let $x = \sqrt{N}$

$x^2 = N$

$x^2 - N = 0$

$f(x) = x^2 - N.$

$f'(x) = 2x$

Formula: $x_{n+1} = x_n - \left[\frac{f(x_n)}{f'(x_n)}\right]$

$$x - \left[\frac{f(x)}{f'(x)}\right] = x - \left[\frac{x^2 - N}{2x}\right] = \frac{2x^2 - x^2 + N}{2x}$$

$$= \frac{x^2 + N}{2x} = \frac{1}{2}\left[\frac{x^2}{x} + \frac{N}{x}\right]$$

$$\boxed{x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]}$$

3. Derive Newton's Algorithm for finding the p^{th} root of a number N.

Ans:

Let $x^p = N$

$x^p - N = 0$

$f(x) = x^p - N$

$f'(x) = px^{p-1}$

Formula: $x_{n+1} = x_n - \left[\frac{f(x_n)}{f'(x_n)}\right]$

$$x - \left[\frac{f(x)}{f'(x)}\right] = x - \left[\frac{x^p - N}{px^{p-1}}\right]$$

$$= \frac{px^p - x^p + N}{px^{p-1}} = \frac{(p-1)x^p + N}{px^{p-1}}$$

$$\boxed{x_{n+1} = \frac{(p-1)x_n^p + N}{px_n^{p-1}}}$$

4. Find an iterative formula to find the reciprocal of a given number $N (N \neq 0)$.

Ans:

Let $x = \frac{1}{N}$

$\frac{1}{x} = N$

$\frac{1}{x} - N = 0$

$f(x) = \frac{1}{x} - N$

$f'(x) = -\frac{1}{x^2}$

Formula:

$x_{n+1} = x_n - \left[\frac{f(x_n)}{f'(x_n)}\right]$

$$x - \left[\frac{f(x)}{f'(x)}\right] = x - \left[\frac{\frac{1}{x} - N}{-\frac{1}{x^2}}\right] = x + \left[\frac{1}{x} - N\right) x^2$$

$$= x + x - Nx^2 = 2x - Nx^2$$

$x_{n+1} = 2x_n - Nx_n^2$

5. For solving a linear system, Compare Gauss elimination method and Gauss-Jordan method.

Ans:

	Gauss Elimination	Gauss Jordan
1.	Coefficient Matrix is transformed into upper triangular Matrix	Coefficient Matrix transformed into Diagonal Matrix
2.	Back substitution Method is used to find unknown	No method needs to find unknowns. Directly we can find.

6. Write the differences between direct and iterative methods for solving system of eqns.

Ans:

	Direct	Indirect
1.	It gives exact value	It gives approximate soln.
2.	Error in a step will not correct by itself	Self correcting method.
3.	Take less time	Time consuming and labourious

7. Write the differences between Gauss Jacobi and Gauss seidel Method.

Ans:

	Gauss seidel	Gauss Jacobi
1.	Converges faster	Converges slowly
2.	Latest values of unknowns used to find next iteration	Previous iteration values are used. ~~to~~

8. Which of the iterative methods for solving linear system of equations converges faster? why?

Ans:

Gauss seidel Method converges faster than Gauss Jacobi, because, in Gauss seidel we use latest values of unknowns we use whereas previous iteration values of unknowns used in Gauss Jacobi

9. Give two direct methods to solve a system of linear equations:

Ans:

1. Gauss elimination method
2. Gauss Jordan Method.

10. Give two Indirect Methods (or) iterative method to solve a system of linear equations

Ans:

1. Gauss Jacobi Method
2. Gauss seidel Method.

11. What is the use of Power method?

Ans: Power method is used to find a dominant (or) numerically largest eigenvalue of a matrix and its corresponding eigen vector.

12. Write the Procedure to find the least eigenvalue using power method.

Ans:

Let A be a given matrix.

1. Find a dominant eigenvalue of A by power method & let it be λ.
2. Let $B = A - \lambda I$
3. Find the dominant eigenvalue of B by power method & let it be λ_1.
4. Least eigenvalue is $\lambda + \lambda_1$.

13. Write down the order of convergence and the condition for convergence of fixed point iteration method (or) $x = g(x)$ method.

Ans:

Order of Convergence is 1.

Condition for Convergence is

$$|g'(x)| < 1 \text{ in } [a, b].$$

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Unit - II

Interpolation and approximation

Lagranges method

1) State Lagranges interpolation formula for unequal intervals

Ans:

$$y = f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} y_1$$

$$+ \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} y_n.$$

2) Distinguish between Newton's divided difference interpolation and Lagranges interpolation.

Ans:

	Newtons divided difference	Lagranges method
1.	Take less time to find the polynomial	Takes more time
2.	Computation is Easy	Computation is complex.

3) Find the second degree polynomial through the points (0,2), (2,1), (1,0) using Lagranges formula.

Ans:

x	0	2	1
y	2	1	0

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$= \frac{(x-2)(x-1)}{(0-2)(0-1)} 2 + \frac{(x-0)(x-1)}{(2-0)(2-1)}(1) + \frac{(x-0)(x-2)}{(1-0)(1-2)} 0$$

$$= (x^2-3x+2) + \frac{1}{2}(x^2-x)$$

$$\boxed{y = \frac{3}{2}x^2 - \frac{7}{2}x + 2}$$

Newton divided difference

4) Show that $\Delta^3_{bcd}(1/a) = -\frac{1}{abcd}$

Ans:

x	y	Δ	Δ^2	Δ^3
a	$\frac{1}{a}$	$\frac{\frac{1}{b}-\frac{1}{a}}{b-a} = \frac{\frac{a-b}{ab}}{b-a}$ $= -\frac{1}{ab}$	$\frac{-\frac{1}{bc}+\frac{1}{ab}}{c-a} = \frac{\frac{-a+c}{abc}}{c-a}$ $= \frac{1}{abc}$	$\frac{\frac{1}{bcd}-\frac{1}{abc}}{d-a}$ $= \frac{\frac{a-d}{abcd}}{d-a}$ $= -\frac{1}{abcd}$
b	$\frac{1}{b}$	$-\frac{1}{bc}$	$\frac{1}{bcd}$	
c	$\frac{1}{c}$	$-\frac{1}{cd}$		
d	$\frac{1}{d}$			

$\therefore \Delta^3_{bcd}(1/a) = -\frac{1}{abcd}$.

5) Show that $[x_0, x_1] = [x_1, x_0]$ in the divided difference

Ans:

$$[x_0, x_1] = \frac{f(x_1)-f(x_0)}{x_1-x_0}.$$

Now $[x_1, x_0] = \frac{f(x_0)-f(x_1)}{x_0-x_1} = \frac{\cancel{-}[f(x_1)-f(x_0)]}{\cancel{-}(x_1-x_0)} = [x_0, x_1]$

6) Find the second divided difference with arguments a, b, c if $f(x) = \frac{1}{x}$.

Ans:

x	$f(x)$	Δ	Δ^2
a	$\frac{1}{a}$		
		$\frac{\frac{1}{b}-\frac{1}{a}}{b-a} = -\frac{1}{ab}$	
			$\frac{-\frac{1}{bc}+\frac{1}{ab}}{c-a} = \frac{\frac{-a+c}{abc}}{c-a} = \frac{1}{abc}$
b	$\frac{1}{b}$		
		$-\frac{1}{bc}$	
c	$\frac{1}{c}$		

Ans: $\frac{1}{abc}$.

7) Construct the divided difference table for the data (0,1), (1,4), (3,40) & (4,85)

Ans:

x	$f(x)$	Δ	Δ^2	Δ^3
0	1	$\frac{4-1}{1-0} = 3$	$\frac{18-3}{3-0} = 5$	$\frac{9-5}{4-0} = 1$
1	4	$\frac{40-4}{3-1} = 18$	$\frac{45-18}{4-1} = 9$	
3	40	$\frac{85-40}{4-3} = 45$		
4	85			

8) State any two properties of divided difference

Ans:

1) n^{th} divided difference of a n^{th} degree polynomial is constant.

2) Divided differences are symmetric in their arguments.

9) State Newtons divided difference formula.

Ans:

$$f(x) = f(x_0) + (x-x_0)\Delta x_0 + (x-x_0)(x-x_1)\Delta^2 x_0 + \cdots$$
$$+ (x-x_0)(x-x_1)\cdots(x-x_{n-1})\Delta^n x_0.$$

Newtons Forward & Backward

10) State Newtons forward & backward formula.

Ans:

Forward formula:

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \cdots + \frac{u(u-1)\cdots(u-(n-1))}{n!}\Delta^n y_0$$

where $u = \frac{x-x_0}{h}$.

Backward formula

$$y = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \cdots, \text{ where } v = \frac{x-x_n}{h}.$$

11) Given. $y_0 = 3, y_1 = 12, y_2 = 81, y_3 = 200, y_4 = 100$, find $\Delta^4 y_0$

Ans:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	3				
		9			
	12		60		
		69		-10	
	81		50		-259
		119		-269	
	200		-219		
		-100			
	100				

$$\Delta^4 y_0 = -259$$

Cubic spline

12) Define cubic spline

Ans:

A cubic polynomial which has continuous slope and curvature is called a cubic spline.

13) What are the advantages of cubic spline

Ans:

Cubic splines provide better approximation for cubic polynomials.

14) Write the conditions (or 4n conditions) for a cubic spline

Ans:

(i) $f(x)$ is a polynomial of degree 1 for $x < x_0$ & $x > x_n$

(ii) $f(x)$ is cubic in (x_{i-1}, x_i), $i = 1, 2, \ldots, n$

(iii) $f(x), f'(x), f''(x)$ are continuous.

(iv) $f(x_i) = y_i$ $i = 0, 1, \ldots, n$.

Extras:

15) Define Interpolation & Extrapolation.

Ans:

Interpolation is a process of finding values between known values.

Extrapolation is a process of finding values beyond known values.

16) Find the divided difference of $f(x) = x^3 + x + 2$ for 1, 3

Ans: when $x = 1$, $f(x) = 4$

$x = 3$, $f(x) = 32$

x	$f(x)$	Δ
1	4	$\frac{32-4}{3-1} = 14$
3	32	

Unit - III

Numerical Differentiation and Numerical Integration

Numerical Integration

1) State Simpson's 1/3 rule.

Ans:

$$\int_a^b f(x)dx = \frac{h}{3}\left[(y_0+y_n)+2(y_2+y_4+y_6+\cdots+y_{n-2}) + 4(y_1+y_3+\cdots+y_{n-1})\right].$$

2) What are the errors in Trapezoidal & Simpson's rule of numerical Integration

Ans:

Trapezoidal rule error term: $E = -\left(\frac{b-a}{12}\right)h^2 y''(\xi)$

Simpson's rule error term: $E < \frac{-h^4}{180}(b-a)y^{iv}(\xi)$

3) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule.

Ans: Take $n=2$, $h = \frac{b-a}{n} = \frac{1-0}{2} = 0.5$, $f(x) = \frac{1}{1+x^2}$.

x	0	0.5	1
$f(x)$	1	0.6666	0.5

By Trapezoidal, $\int_a^b f(x)dx = \frac{h}{2}\left[(y_0+y_2)+2(y_1)\right]$

$$\int_0^1 \frac{dx}{1+x^2} = \frac{0.5}{2}\left[(1+0.5)+2(0.6666)\right] = 0.7083.$$

4) Compare Trapezoidal rule and Simpson's 1/3 rule for evaluating numerical Integration.

Ans:

Ans:

	Trapezoidal	Simpson's 1/3 rule
1.	no. of intervals 'n' may be any number.	n must be even
2.	Degree is one	Degree is two
3.	Error ~~(E)~~ $E = -\left(\frac{b-a}{12}\right)h^2 y''(\xi)$	$E = \frac{b-a}{180} h^4 y''(\xi)$

5) State Gaussian 2-point & 3-point Quadrature formula

Ans:

Gaussian 2-pt formula

$$\int_{-1}^{1} f(x)dx = f\left(-\sqrt{\tfrac{1}{3}}\right) + f\left(\sqrt{\tfrac{1}{3}}\right)$$

Gaussian 3-pt formula.

$$\int_{-1}^{1} f(x)dx = \frac{5}{9}\left[f\left(-\sqrt{\tfrac{3}{5}}\right) + f\left(\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9} f(0)$$

6) Change the limits into $(-1,1)$ from $\int_0^{\pi/2} \sin x\, dx$

Ans: Put $x = \left(\frac{b-a}{2}\right)t + \left(\frac{b+a}{2}\right)$

Here $a = 0$ $b = \pi/2$.

$$\Rightarrow x = \left(\frac{\pi/2 - 0}{2}\right)t + \left(\frac{\pi/2 + 0}{2}\right) \text{ i.e., } x = \pi/4\, t + \pi/4.$$

$$dx = \pi/4\, dt.$$

$$\therefore \int_0^{\pi/2} \sin x\, dx = \int_{-1}^{1} \sin\left(\pi/4\, t + \pi/4\right) \pi/4\, dt.$$

7) Apply two point Gaussian Quadrature formula to evaluate $\int_{-1}^{1} \frac{1}{1+x^2} dx$.

Ans: Formula

$$\int_{-1}^{1} f(x) dx = f\left(-\sqrt{\tfrac{1}{3}}\right) + f\left(\sqrt{\tfrac{1}{3}}\right)$$

Here $f(x) = \frac{1}{1+x^2}$. & $f\left(-\frac{1}{\sqrt{3}}\right) = 0.75$

$f\left(\frac{1}{\sqrt{3}}\right) = 0.75$

$$\therefore \int_{-1}^{1} \frac{1}{1+x^2} dx = 0.75 + 0.75$$

$$= 1.5.$$

Numerical differentiation

8). write down the expression for $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ at $x = x_n$ by Newton's backward difference formula.

Ans

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$$

9) write the formula for $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ at $x = x_0$ by Newton's forward difference formula.

Ans:-

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\right)$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \cdots\right]$$

Unit - IV

Initial value Problem for Ordinary diff. eqn.

Taylor series method.

1. State the advantages of R-K method over Taylor series method.

Ans:

(i) R-K method do not need higher order derivatives to solve the problem.

(ii) Accuracy in R-K method is higher than Taylor's method

2. State the advantage & disadvantages of Taylor series method

Ans:

Advantage : This method is very powerful to find the initial values for methods like Adams & Milne's.

Disadvantage (Drawback) : Calculation of higher order derivatives are tedious when the pbm is complicated.

3. Find $y(1.1)$ if $y' = x+y$, $y(1) = 0$ by Taylor series method.

Ans: Gu. $x_0 = 1$, $y_0 = 0$, $h = 0.1$

$y' = x+y \qquad y_0' = x_0 + y_0 = 1$

$y'' = 1+y' \qquad y_0'' = 1 + y_0' = 2$

$y''' = y'' \qquad y_0''' = y_0'' = 2.$

$$y_1 = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0'''$$

$$= 0 + 0.1(1) + \frac{(0.1)^2}{2} 2 + \frac{(0.1)^3}{6} 2$$

$$\boxed{y_1 = 0.1103}$$

Euler's & Modified Euler's method

4) State Euler's formula for solving $y' = f(x,y)$, $y(x_0) = y$

Ans:

$$y_{n+1} = y_n + h f(x_n, y_n), \quad n = 0, 1, 2, \ldots$$

5) State the modified Euler formula to find $y(x_1)$ for solving $\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0$.

Ans:

$$y_{n+1} = y_n + h\left[f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right)\right], \quad n = 0, 1, 2, \ldots$$

(or) $$y_1 = y_0 + h\left[f\left(x_0 + h/2, y_0 + h/2\, f(x_0, y_0)\right)\right].$$

6) Using Euler's method, find $y(0.2)$, $y(0.4)$ from $y' = x+y$, $y(0) = 1$.

Ans: Gn. $y' = x+y$

(or) $f(x,y) = x+y$, $h = 0.2$, $x_0 = 0$, $y_0 = 1$.

$x_1 = 0.2$, $y_1 = ?$

$x_2 = 0.4$, $y_2 = ?$

$$y_{n+1} = y_n + h f(x_n, y_n)$$

$$y_1 = y_0 + h f(x_0, y_0)$$
$$= 1 + (0.2)(0 + 0.2)$$
$$= 1.04.$$

$$y_2 = y_1 + h f(x_1, y_1)$$
$$= 1.04 + 0.2(0.2 + 1.04)$$
$$= 1.288.$$

Multi Step Methods

7) Compare Single - step & Multi Step Method

Ans:

	Single Step Method	Multi Step Method
1.	Only one prior value is needed	Four prior values are needed
2.	No information about truncation error	Easily get good estimation of truncation error.

8). How many prior values are require to predict the next value in Multi step methods (or) Milne's (or) Adam's method?

Ans:

Four prior values.

9) State Milne's predictor - corrector formula.

Ans: Predictor formula

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

Corrector formula

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right].$$

10) State Adam-Bashforth formula

Ans:

Predictor formula

$$y_{n+1,p} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right]$$

Corrector formula

$$y_{n+1,c} = y_n + \frac{h}{24}\left[9y_{n+1}' + 19y_n' - 5y_{n-1}' + y_{n-2}'\right].$$

Unit - V

Boundary Value Problems in Ordinary & Partial diff Eqns:

Finite difference method

1) Write the central difference approximation of $y''(x)$ & $y'(x)$

(or)

Write down the finite difference formula for $y'(x)$ & $y''(x)$

Ans:-

$$y_i' = \frac{1}{2h}\left[y_{i+1} - y_{i-1}\right].$$

$$y_i'' = \frac{1}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right].$$

2) Obtain the finite difference scheme for the differential eqn. $2y''(x) + y(x) = 5$ (or) $2y'' + y = 5$

Ans:

$$2y_i'' + y_i = 5$$

$$y_i'' = \frac{1}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right]$$

$$2\left[\frac{1}{h^2}\left(y_{i-1} - 2y_i + y_{i+1}\right)\right] + y_i = 5$$

$$\frac{2y_{i-1} - 4y_i + 2y_{i+1} + 2y_ih^2}{h^2} = 5$$

$$2y_{i+1} + y_i\left[2h^2 - 4\right] + 2y_{i-1} = 5h^2.$$

~~Multi Step Methods~~

Classification of PDE.

3) Classify the following equation $u_{xx}+4u_{xy}+4u_{yy}-u_x+2u_y=0$

Ans:

$A=1 \quad B=4 \quad C=4.$

$B^2-4AC = 4^2-4\cdot1\cdot4 = 16-16=0$

$B^2-4AC=0 \quad \therefore$ It is parabolic.

Note:

If $B^2-4AC>0$, hyperbolic

$B^2-4AC<0$, Elliptic

4) Classify the PDE $\dfrac{\partial^2 u}{\partial x^2}-2\dfrac{\partial^2 u}{\partial x\partial y}+\dfrac{\partial^2 u}{\partial y^2}=0.$

Ans: $u_{xx}-2u_{xy}+u_{yy}=0.$

$A=1,\ B=-2,\ C=1$

$B^2-4AC=(-2)^2-4(1)(1)$

$=4-4=0$, Parabolic.

5) Classify the PDE $u_{xx}=u_t.$

Ans:

$u_{xx}-u_t=0$

$A=1, B=0, C=0$

$B^2-4AC = 0-4(1)(0)=0$, Parabolic.

One dimensional Heat eqn.

6) Write any two methods to solve one dimensional heat eqn.

Ans:-

1) Bender-schmidt relation (Explicit scheme).

2) Crank-Nicholson scheme (Implicit scheme)

7) State whether the Crank-Nicholson's scheme is an explicit (or) implicit scheme. Justify.

Ans:

It is an Implicit scheme, because it does not give the value of u at $t = t_{j+1}$ in terms of values of u at $t = t_j$ directly.

8) Write down the explicit finite difference method for solving one dimensional wave eqn.

Ans:

$$u_{i,j+1} = \lambda u_{i+1,j} + (1-2\lambda) u_{i,j} + \lambda u_{i-1,j}$$

where $\lambda = \frac{k}{ah^2}$ & is valid in $0 < \lambda \leq 1/2$.

If $\lambda = 1/2$, $u_{i,j+1} = \frac{1}{2}\left[u_{i-1,j} + u_{i+1,j}\right]$ & is called Bender-schmidt relation.

Note: Qn 8, can also be asked as,

write down Bender-schmidt difference scheme in general form and using suitable value of λ, write the scheme in simplified form.

9) State Crank-Nicholson's difference scheme.

Ans:

$$(2\lambda+2)u_{i,j+1} = \lambda\left[u_{i-1,j}+u_{i+1,j}+u_{i-1,j+1}+u_{i+1,j+1}\right]+(2-2\lambda)u_{i,j}$$

where $\lambda = \frac{k}{ah^2}$.

If $\lambda=1$, $u_{i,j+1} = \frac{1}{4}\left[u_{i-1,j}+u_{i+1,j}+u_{i-1,j+1}+u_{i+1,j+1}\right]$.

One dimensional wave eqn:

10) State the finite difference scheme to solve the eqn $y_{tt} = \alpha^2 y_{xx}$

(or)

Express in $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ in terms of difference approximat

Ans:

$$u_{i,j+1} = 2(1-\lambda^2a^2)u_{i,j}+\lambda^2a^2(u_{i-1,j}+u_{i+1,j})-u_{i,j-1}$$

where $\lambda = \frac{k}{h}$.

If $k=\frac{h}{a}$, $u_{i,j+1} = u_{i-1,j}+u_{i+1,j}-u_{i,j-1}$.

11) Write down the explicit formula to solve the hyperbolic equation $u_{tt}=9u_{xx}$ when $\Delta x=0.25$ & $\Delta t=1/16$.

Ans: Here $h=0.25$ $k=1/16$ $a^2=9$ $a=3$.

Now $\lambda = \frac{k}{h} = \frac{\frac{1}{16}}{0.25} = \frac{1}{4}$.

Formula

$$u_{i,j+1} = 2(1-\lambda^2a^2)u_{i,j}+\lambda^2a^2(u_{i-1,j}+u_{i+1,j})-u_{i,j-1}.$$

$$u_{i,j+1} = 2\left(1-\tfrac{1}{16}\cdot 9\right)u_{i,j}+\tfrac{1}{16}\cdot 9\left(u_{i-1,j}+u_{i+1,j}\right)-u_{i,j-1}$$

(or) $$u_{i,j+1} = \frac{7}{8}u_{i,j}+\frac{9}{16}\left[u_{i-1,j}+u_{i+1,j}\right]-u_{i,j-1}.$$

Laplace & Poisson eqn.

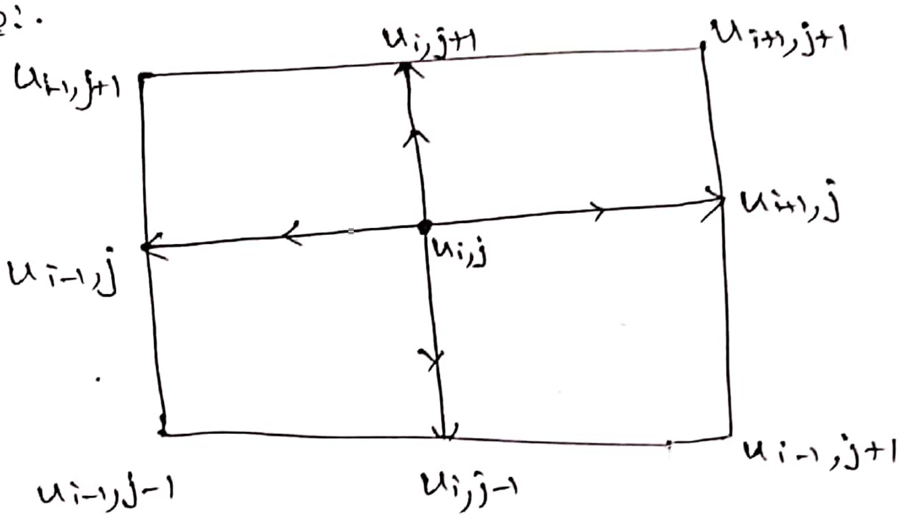
12) Write Liebman's iteration process.

Ans:

$$u_{i,j}^{(n+1)} = \frac{1}{4}\left[u_{i-1,j}^{(n+1)} + u_{i,j+1}^{(n+1)} + u_{i+1,j}^{(n)} + u_{i,j-1}^{(n)}\right].$$

13) Write down the standard five point formula to find the numerical soln of Laplace eqn.

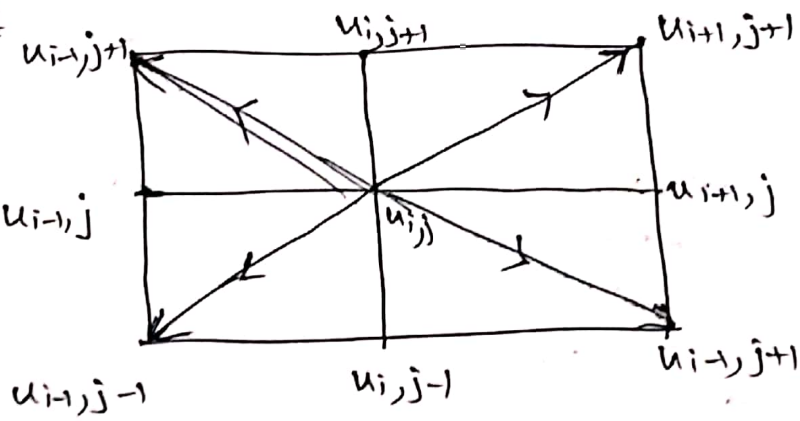
Ans:



$$u_{i,j} = \frac{1}{4}\left[u_{i-1,j} + u_{i,j+1} + u_{i+1,j} + u_{i,j-1}\right].$$

14) Write the Diagonal five point formula to solve $\nabla^2 u = 0$.

Ans



$$u_{i,j} = \frac{1}{4}\left[u_{i-1,j+1} + u_{i+1,j+1} + u_{i-1,j-1} + u_{i-1,j+1}\right].$$

15) Write down the difference scheme for solving the Poisson equation $\nabla^2 u = f(x,y)$

Ans:-

$$u_{i-1,j} + u_{i,j+1} + u_{i+1,j} + u_{i,j-1} - 4u_{i,j} = h^2 f(ih, jh).$$